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COMMENTS ON 'A SIMPLE PROOF OF THE SEPARATION THEOREM FOR LINEA--ETC(U)

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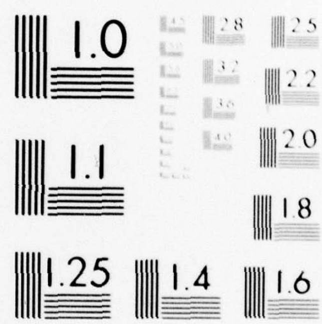

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFOSR-TR- 79-0699	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) COMMENTS ON "A SIMPLE PROOF OF THE SEPARATION THEOREM FOR LINEAR STOCHASTIC SYSTEMS WITH TIME DELAYS"	5. TYPE OF REPORT & PERIOD COVERED Interim
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) ANDERS LINDQUIST	8. CONTRACT OR GRANT NUMBER(s) (15) AFOSR-78-3519
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Kentucky Department of Mathematics Lexington, KY 40506	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F (16) 2304/A1 (17) A1
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, Washington, D.C. 20332	12. REPORT DATE (11) 1979 (12) 6p.
	13. NUMBER OF PAGES 4
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

These comments were provided to illustrate a probable misunderstanding in trying to prove the separation theorem for linear stochastic systems with time delays. The author points outs specific problems and amends the proof to show a better method.

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COMMENTS ON "A SIMPLE PROOF OF THE SEPARATION THEOREM FOR LINEAR STOCHASTIC SYSTEMS WITH TIME DELAYS"

Anders Lindquist

The above mentioned proof[1] contains a subtle circular argument, which is rather common in the literature; it can, for example, also be found in [3][2]. A few comments are therefore in order.

The authors consider a linear stochastic system with time delays having state process x, input (control) process u and output process z. (The fact that there are time delays in the system is not important here.) The problem is to devise a nonrandom feedback loop

$$u(t) = \pi(t, z(s);\ 0 \leq s \leq t)$$

so that the resulting feedback equations have a unique solution and a quadratic criterion is minimized. To emphasize the dependence of x and z on the control process u, we shall here write x_u and z_u. Using the transition matrix, the state equation can be integrated to attain the form

$$x_u(t) = x_o(t) + \int_0^t K(t,s)u(s)\, ds$$

The author is with the Department of Mathematics, University of Kentucky, Lexington, Kentucky 40506. This work was supported by the Air Force Office of Scientific Research under grant AFOSR-78-3519.

[1] R. E. Schmotzer and G. L. Blankenship, *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 4, August 1978, pp.734-735.

[2] All reference numbers refer to the references in the correspondence item[1].

where K is a matrix kernel and the index o indicates that no control is applied. Given any two controls u and v, define

$$\hat{x}_{u|v}(s|t) = E\{x_u(s) \mid z_v(\tau);\ 0 \le \tau \le t\}.$$

The proof[1] is based on a simple completion-of-the-square argument adapted from [3] which requires that, for all $s \le t$, the estimation error

$$\tilde{x}_{u|u}(s|t) = x_u(s) - \hat{x}_{u|u}(s|t)$$

is independent of the choice of control signal u. The proof of this fact, however, is based on the assumption that

$$\hat{x}_{u|u}(s|t) = \hat{x}_{0|0}(s|t) + \int_0^s K(s,t)u(\tau)\, d\tau$$

for $s \le t$ (where $\hat{x}_{0|0}$ is the estimate obtained when there is no control), which in turn is equivalent to $\tilde{x}_{u|u}$ being independent of the control, thus closing the circle. In fact, *a priori* we know only that

$$\hat{x}_{u|u}(s|t) = \hat{x}_{0|u}(s|t) + \int_0^s K(s,\tau)u(\tau)\, d\tau,$$

and to show that $\hat{x}_{0|u} = \hat{x}_{0|0}$ we must first prove that the sigma-fields Z_t^u generated by $\{z_u(\tau);\ 0 \le \tau \le t\}$ for each t are independent of the choice of control law.

This is by no means a trivial question. The control dependence can be eliminated by applying the Girsanov transformation, but this implies that the solution of the resulting system equations exists only in a weak sense, and consequently some of the physical meaning of the feedback problem is lost. Hence we shall only consider strong solutions. As explained in [10], this requires some care in defining the class of admissible control laws.

There are several ways to modify the note[1] so that the proof becomes correct. The simplest is to allow only linear feedback laws, i.e., π belonging to the class L defined by

$$\pi(t,z) = f(t) + \int_0^t F(t,s)\, dz(s),$$

where f is a vector function and F a matrix function, both square-integrable. It is not hard to prove [10] that the sigma-fields $\{Z_t^u\}$ are constant in this case.

Although the candidate π^* for optimal control law (see equ. 4 in the note[1]) belongs to L, we usually want to know how it compares with nonlinear feedback controls. As in [11], to insure the sigma-field constancy, we can require that π be a Lipschitz continuous function of the sample functions of z, but we must realize that L is not contained in this class of control laws, the controls of L not being defined samplewise unless the functions $s \mapsto F(t,s)$ have bounded variation. Hence we must impose some technical assumptions to insure that π^* is admissible.

A more exhaustive class of nonlinear feedback controls can be defined by first observing that the sigma-field constancy problem does not occur when there is a positive delay in the feedback loop, i.e., the control law π is of the form

$$u(t) = \pi(t, z(s);\ 0 \le s \le t - \varepsilon)$$

where $\varepsilon > 0$, and then noting that properly defined limits of such π will also do. (The limit in probability of a sequence of random vectors will retain the measurability property of the sequence.)

Finally, in the proof[1] it is necessary to assume that the system is *Gaussian*, i.e., that the driving noises *and* the initial conditions are (jointly) Gaussian; otherwise the estimates $\hat{x}_{0|0}$ will not be linear in the data as required. If all conditional expectations are replaced by wide sense conditional expectations, the Gaussian assumption can be dispensed with *provided that* the analysis is restricted to the linear class L; this will not work for nonlinear control laws. Removing the Gaussian condition in the general case will lead to nonlinear filtering. Formally the optimal solution will be the same function of the state estimate $\hat{x}$ as in the Gaussian case, but it will probabily be hard to show that the corresponding feedback law π is admissible, i.e., that the feedback system has a unique (strong) solution and that the sigma-fields $\{Z_t^u\}$ are constant.